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2005 J. Phys. A: Math. Gen. 38 1225

(http://iopscience.iop.org/0305-4470/38/6/003)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **38** (2005) 1225–1233 doi:10.1088/0305-4470/38/6/003

The one-loop elastic coefficients for the Helfrich membrane in higher dimensions

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Received 26 May 2004, in final form 25 November 2004
Published 26 January 2005
Online at stacks.iop.org/JPhysA/38/1225

Abstract
Using a covariant geometric approach we obtain the effective bending couplings for a two-dimensional rigid membrane embedded into a $(2 + D)$-dimensional Euclidean space. The Hamiltonian for the membrane has three terms: the first one is quadratic in its mean extrinsic curvature. The second one is proportional to its Gaussian curvature and the last one is proportional to its area. The results we obtain are in agreement with those finding that thermal fluctuations soften the two-dimensional membrane embedded into a three-dimensional Euclidean space.

PACS numbers: 87.16.Dg, 46.70.Hg

1. Introduction

Because of their chemical structure, when one adds amphiphilic molecules to an aqueous substance they spontaneously form membrane structures known as micelles of various geometrical shapes (spherical or cylindrical in particular). The relevance of these structures, in part, is that they can be regarded as simplified models of biological molecules and that they can actually be created artificially in the laboratory for its use in cosmetics and detergents [1]. Despite the diversity of these amphiphilic membranes, they can be modelled by a single geometric Hamiltonian that captures their relevant physical properties as the surface tension μ and rigidity κ. For the last, a term that penalizes the energetic cost of giving curvature to the membrane has been suggested [2, 3]. Surprisingly, geometric Hamiltonians of this sort of surfaces appear in several areas of physics ranging from string theory and cosmology to condensed matter [4–7].

In equilibrium, the membrane configurations satisfy the Euler–Lagrange equations which are obtained by extremizing the Hamiltonian with respect to the membrane orthogonal deformations. These equations are essentially geometric in their own nature. The evaluation of the second variation of the Hamiltonian is necessary to study the stability of equilibrium solutions and to understand the effect of fluctuations on the shape of the membrane. If one wishes to understand, for instance, the transitions between different shapes that the membrane can have, one has to study the effect of small fluctuations on the membrane shape. A problem in statistical physics, in this context, is to study the membrane elasticity under thermal fluctuations, i.e., to investigate the effect of such fluctuations on the bending constants, the membrane rigidity in particular [8–10]. This problem has already been considered by a number of authors [11–15]. The conclusion they reach is that thermal fluctuations soften the membrane. For a quasi-two-dimensional membrane there exists a non-trivial fixed point T_c: a membrane with temperature lower than T_c is very rigid at long length scales, but one with temperature greater than T_c has very low rigidity (becoming zero for length scales beyond a characteristic length called the persistence length). For exactly two-dimensional membranes, which is the critical dimension of the model, it can be interpreted that the membrane disappears as a consequence of fluctuations. This is similar to what occurs in the $O(N)$ nonlinear sigma model where, in two dimensions, fluctuations destroy any attempt at ordering. Recently, however, it has been suggested that thermal fluctuations rather stiffen the membrane [16, 17] instead of softening it. The interpretation of this result is not clear yet. So, because of this, we have considered it useful to investigate the subject using an alternative approach.

In principle, there are two different approaches which allow one to study the effect of thermal fluctuations on the membrane. One of them, that we use here, consists of a perturbative expansion for finite D codimension with the inverse of the rigidity T/κ (T the temperature) being the perturbative parameter. Our results indicate that thermal fluctuations lower the value of the rigidity, so the model falls into the two-dimensional membrane scenario described above. The other approach consists of a non-perturbative approximation in D within the $D \to \infty$ limit [18]. As pointed out in [19], and in analogy with the $O(N)$ nonlinear sigma model, the non-perturbative $D \to \infty$ limit approximation must somehow regain the properties of a two-dimensional membrane in a three-dimensional Euclidean space within the one-loop approximation. In particular, the one-loop result for the membrane rigidity of codimension D is exact within the $D \to \infty$ limit [18]. This gives rise to the question whether this is still valid to order two loops. The approach we propose in this paper may, amongst other things, give an answer to this question or it can help to understand, for instance, the effect of quantum fluctuations in geometric models from string theory as those proposing the rigidity term as fundamental for their description [20].

To be explicit, the work in this paper consists in obtaining, in the one-loop order, the elastic bendings for a two-dimensional membrane embedded into a $(2 + D)$-dimensional Euclidean space. Though it is possible to generalize for higher-dimensional membranes, we consider this the most relevant case. The calculation of the elastic bendings, to this order, starts by determining the operator of fluctuations associated with small deformations in the shape of the membrane. A common approach for that is to consider the Monge representation where the Hamiltonian is expanded in terms of the membrane height, $z = h(x, y)$. Beyond the Gaussian model, perturbation theory yields autointeraction terms extremely difficult to deal with. It is important to realize, however, that carrying out a perturbative expansion can be advantageous indeed as it accounts for the crumpling transition [10]. There is the disadvantage, though, of losing covariance in the expressions appearing in intermediate calculations.

For this investigation we use an alternative formalism, capable of dealing with geometric models such as the Helfrich model for fluid membranes, which does preserve covariance in all expressions. This formalism has been extensively used before by Capovilla and Guven [21], and is found to be particularly powerful in simplifying long calculations. As an example, within this formalism the Euler–Lagrange equations can be obtained in a few lines from a geometric Hamiltonian. Moreover, this fully covariant approach allows one to solve several technically difficult problems such as the description of lipid membranes with an edge and to tackle the problem of membrane adhesion [22, 23]. In addition to these, it has recently been used to investigate thermal fluctuations of fluid membranes in higher dimensions. As a result, the operator of fluctuations $\mathcal{L}^{ij}$ for a N-dimensional membrane embedded into an Euclidean space of dimension $(N + D)$ has been found in a systematic manner [24].

The Hamiltonian associated with the model under study is written as a sum of three terms: the first one is quadratic in the mean extrinsic curvature, K^i. The second one is proportional to the scalar curvature of the membrane $\mathcal{R}$ and the last one proportional to its area A,

$$\mathcal{H}_0 = \frac{\kappa}{2}\int \mathrm{d}A\, K^i K_i + \beta \int \mathrm{d}A\, \mathcal{R} + \mu \int \mathrm{d}A. \tag{1}$$

Given that $[K] = L^{-1}$, any other terms of higher order in the curvature are irrelevant for two-dimensional membranes in the long-length regime. This $\mathcal{H}_0$ above is a generalized model of a lipid membrane with non-zero rigidity and surface tension which has also been studied in the string theory context [25, 26]. The model is invariant under translations and rotations in space. The partition function Z is constructed as a functional integral over all possible surface configurations, $\mathbf{X}$, of the form

$$Z = \int \mathcal{D}\mathbf{X} \exp(-\mathcal{H}_0/T), \tag{2}$$

where T is the temperature parameter. In the one-loop order, we expand up to second order the exponent in equation (2) around a configuration extremizing $\mathcal{H}_0$. Note that, as discussed in [27], even in the one-loop order the integral measure contributes with a nonlinear factor (a sort of Liouville term) that has to be taken into account. Leaving this for later consideration and after computing a Gaussian integral, the one-loop effective Hamiltonian may be formally written as

$$\mathcal{H}_{\mathrm{eff}} = \mathcal{H}_0 + \frac{T}{2}\,\mathrm{Tr}\log \mathcal{L}^{ij}, \tag{3}$$

with $\mathcal{L}^{ij}$ the operator for small fluctuations and Tr representing the functional trace. Subsequently, we obtain the one-loop effective bending coefficients of the model and present the results below. As will be shown, they are in agreement with those obtained for the case of a two-dimensional membrane embedded into a three-dimensional Euclidean space.

The organization of this paper is as follows. As the fundamental tool used throughout this work is essentially geometry, we describe in section 2 the geometry of embedded surfaces. In section 3 we study deformations in the quantities present in the Hamiltonian. Further, the Euler–Lagrange equations of the problem are also written here. In section 3.2 we present the operator of fluctuations for the Hamiltonian and in section 4 we obtain the one-loop effective parameters. Finally, our results are summarized in section 5.

2. Geometry and deformations

An orientable N-dimensional surface Σ, embedded into the $(N + D)$-dimensional Euclidean space R^{N+D}, is locally specified by the embedding functions,

$$\mathbf{x} = \mathbf{X}(\xi^a), \tag{4}$$

where $\mathbf{X} = (X^1, \ldots, X^{N+D})$ are $(N + D)$ functions of N variables each. The metric γ_{ab}, induced on Σ, is defined as

$$\gamma_{ab} := \mathbf{e}_a \cdot \mathbf{e}_b, \tag{5}$$

with $\mathbf{e}_a = \partial_a \mathbf{X}\,(\partial_a = \partial/\partial\xi^a)$ and we are using the inner product in R^{N+D}. This metric γ_{ab} determines the intrinsic geometry of Σ.

The Riemann curvature $\mathcal{R}^a{}_{bcd}$ is defined as

$$(\nabla_a\nabla_b - \nabla_b\nabla_a)V^c = \mathcal{R}^c{}_{dab}V^d, \qquad (6)$$

with ∇_a being the covariant derivative associated with γ_{ab}. The Ricci tensor is defined by $\mathcal{R}^{ab} := \mathcal{R}^c{}_{acb}$ and the scalar curvature given by $\mathcal{R} := \gamma_{ab}\mathcal{R}^{ab}$.

Let us now examine the extrinsic geometry of Σ. The D unit vectors $\mathbf{n}_i(\xi)$, normal to Σ in R^{N+D}, are defined implicitly by

$$\mathbf{e}_a \cdot \mathbf{n}^i = 0, \qquad (7)$$

which, after normalization, satisfy

$$\mathbf{n}^i \cdot \mathbf{n}^j = \delta^{ij}. \qquad (8)$$

Note that these equations determine the normals $\mathbf{n}^i$ up to an arbitrary rotation. Therefore, the definition is invariant under local rotations,

$$\mathbf{n}^i \to O^i{}_j(\xi^a)\mathbf{n}^j. \qquad (9)$$

The set of vectors $\{\mathbf{n}^i, \mathbf{e}_a\}$ is precisely an adapted base on the surface. Their gradients along the surface are written in terms of this base as the Gauss–Weingarten equations

$$\partial_a\mathbf{e}_b = \gamma_{ab}{}^c\mathbf{e}_c - K^i_{ab}\mathbf{n}_i, \qquad (10)$$

$$\partial_a\mathbf{n}_i = K_{ab\,i}\gamma^{bc}\mathbf{e}_c + \omega_{ij\,a}\mathbf{n}^j. \qquad (11)$$

The extrinsic curvatures of Σ are given by the D symmetric rank-two surface tensors, $K_{abi} := -\mathbf{n}_i \cdot \partial_a\mathbf{e}_b = K_{bai}$. We also define the trace with respect to the induced metric,

$$K_i := \gamma^{ab}K_{abi}. \qquad (12)$$

Both the intrinsic and extrinsic geometry of Σ, which are determined by γ_{ab}, $K_{ab\,i}$ and ω_{ija}, can no longer be specified independently. They are related by integrability conditions that correspond to generalized Gauss–Codazzi and Codazzi–Mainardi equations, and also by equations that fix the curvature Ω_{ijab} associated with the $SO(D)$ connection ω_{ija} in terms of K^i_{ab}:

$$\mathcal{R}_{abcd} - K_{ac\,i}K^i_{bd} + K_{ad\,i}K^i_{bc} = 0, \qquad (13)$$

$$\tilde{\nabla}_a K^i_{bc} - \tilde{\nabla}_b K^i_{ac} = 0, \qquad (14)$$

$$\Omega_{ij\,ab} + K_{ac\,i}K^c{}_{b\,j} - K_{ac\,j}K^c{}_{b\,i} = 0. \qquad (15)$$

In these expressions $\tilde{\nabla}_a$ is the $SO(D)$ covariant derivative defined as

$$\tilde{\nabla}_a\phi_i = \nabla_a\phi_i - \omega_{ij\,a}\phi^j. \qquad (16)$$

3. Deformations in the geometry

3.1. The metric and the extrinsic curvature

We proceed as in the hypersurface case and project the deformation onto its normal and tangent components in the form [28]

$$\delta\mathbf{X} = \Phi^i\mathbf{n}_i + \Phi^a\mathbf{e}_a. \qquad (17)$$

By now we will assume that the surface has no boundary so that the tangent deformation can be associated with a reparametrization. The only physical deformations are those normal to the surface. Let us then examine the deformation in the adapted base $\{\mathbf{e}_a, \mathbf{n}^i\}$ under these normal deformations. Therefore, the relationships

$$\delta_\perp \mathbf{e}_a = (\tilde{\nabla}_a \Phi^i)\mathbf{n}_i + \Phi^i K_{ia}{}^c \mathbf{e}_c, \tag{18}$$

$$\delta_\perp \mathbf{n}^i = -(\tilde{\nabla}^a \Phi^i)\mathbf{e}_a, \tag{19}$$

hold. In these we have used the covariant derivative $\tilde{\nabla}$ defined in equation (16). By using equation (18), it is straightforward to show that the deformation in the induced metric is given by

$$\begin{aligned} \delta_\perp \gamma_{ab} &= (\delta_\perp \mathbf{e}_a) \cdot \mathbf{e}_b + \mathbf{e}_a \cdot (\delta_\perp \mathbf{e}_b) \\ &= 2K_{iab}\Phi^i. \end{aligned} \tag{20}$$

From this, we get the first-order deformation of the infinitesimal area element

$$\delta_\perp \, \mathrm{d}A = \mathrm{d}A\, K^i \Phi_i. \tag{21}$$

Let us now turn to the first-order deformation of the extrinsic curvature along the *i*th normal vector. For that we find

$$\begin{aligned} \delta_\perp K^i_{ab} &= -(\delta_\perp \mathbf{n}^i) \cdot \partial_a \mathbf{e}_b - \mathbf{n}^i \cdot \partial_a \delta_\perp \mathbf{e}_b \\ &= -\nabla_a \nabla_b \Phi^i + K_{ac}{}^i K^c{}_{bj} \Phi^j. \end{aligned} \tag{22}$$

As a result, the deformation in the mean extrinsic curvature is

$$\delta_\perp K^i = -\tilde{\Delta}\Phi^i - K^i_{ab} K^{ab}{}_j \Phi^j. \tag{23}$$

From these expressions it is straightforward to find the corresponding Euler–Lagrange equations of the model ($\mathcal{E}^j = 0$), which are given by [24]

$$\mathcal{E}^j = -\kappa \tilde{\Delta} K^j + \frac{\kappa}{2}(2R^{ij} - K^i K^j)K_i - 2\beta \mathcal{G}_{ab} K^{abj} + \mu K^j. \tag{24}$$

3.2. Second-order deformations

To second order we express the result in terms of the deformations already in hand. Therefore, given the Hamiltonian in equation (1), we find that the operator of fluctuations can be written in the compact form

$$\mathcal{L}_{ij} = \kappa \tilde{\Delta}^2_{ij} + A^{abik}(\tilde{\nabla}_a \tilde{\nabla}_b)_{kj} + B^a_{ik}(\tilde{\nabla}_a)_{kj} + C_{ij}, \tag{25}$$

with the coefficient of the Laplacian term given by ($K^2 = K_i K^i$)

$$\begin{aligned} A^{abij} = {}& \frac{\kappa}{2}(2K^i K^j - K^2\delta^{ij} - 4R^{ij})\gamma^{ab} - \mu\delta^{ij}\gamma^{ab} \\ & + 2\kappa K_\ell K^{\ell ab}\delta^{ij} + 2\beta(\mathcal{G}^{abji} + \mathcal{G}^{baij} + \mathcal{G}^{ab}\delta^{ij}); \end{aligned} \tag{26}$$

where the definitions

$$\mathcal{G}_{ab}{}^{ij} = R_{ab}{}^{ij} - \frac{\gamma_{ab}}{2} R^{ij}, \tag{27}$$

$$R_{ab}{}^{ij} = K^i_{ab} K^j - K_a{}^{ci} K_{cb}{}^j, \tag{28}$$

have been used. Both coefficients $\mathcal{B}$ and $\mathcal{C}$ contain $K\nabla K$ and K^4 terms respectively [24]; which will modify, in any case, terms of higher order in the membrane curvature. These, however, have not been taken into account in the Hamiltonian because they are irrelevant for long-length scales for two-dimensional membranes.

4. The heat kernel regularization

The heat kernel provides a covariant method for calculating the short distance behaviour of the effective Hamiltonian [29–31]. One examines the heat equation associated with the Laplacian on the surface. Its kernel satisfies

$$\left(\frac{\partial}{\partial s} + \tilde{\Delta}\right) \mathcal{K}_{ij}(\xi, \xi', s) = 0, \tag{29}$$

subject to the initial condition

$$\mathcal{K}_{ij}(\xi, \xi', 0) = \frac{\delta(\xi, \xi')}{\sqrt{\gamma}} \delta_{ij}. \tag{30}$$

The solution to this equation can be written in the form

$$\mathcal{K}^{ij}(\xi, \xi', s) = \frac{1}{(4\pi s)^{N/2}} \exp\left(-\frac{\sigma(\xi, \xi')}{2s}\right) \mathcal{W}^{ij}(\xi, \xi', s) \mathcal{D}^{1/2}(\xi, \xi'), \tag{31}$$

with $\sigma(\xi, \xi')$ being the world function (equal to half the squared geodesic distance between the points ξ and ξ') and $\mathcal{D}(\xi, \xi')$ the Van Vleck–Morette determinant. In the $s \to 0$ limit, the exponential normalized by $1/(4\pi s)^{N/2}$ reduces to the delta function. We thus have that $\mathcal{W}$ is a regular function of its arguments which may be expanded in power series as

$$\mathcal{W}^{ij}(\xi, \xi', s) = \sum_{l=0}^{\infty} s^l a_l^{ij}(\xi, \xi'). \tag{32}$$

In the coincidence limit, $\xi' \to \xi$, the first three coefficients $a_l^{ij}(\xi)$ are given by

$$a_0^{ij} = \delta^{ij}, \quad a_1^{ij}(\xi) = \frac{\mathcal{R}}{6} \delta^{ij}, \tag{33}$$

$$a_2^{ij}(\xi) = \left(\frac{1}{180}\left(\mathcal{R}_{abcd}^2 - \mathcal{R}_{ab}^2\right) + \frac{\mathcal{R}^2}{72} + \frac{\Delta\mathcal{R}}{30}\right) \delta^{ij} + \frac{1}{2} \Omega^i{}_k{}^{ab} \Omega^{kj}{}_{ab}. \tag{34}$$

Henceforth we will absorb the determinant $\mathcal{D}^{1/2}(\xi, \xi')$ into the definition of the a_l'. Despite the undifferentiated coincidence limits being the same, one may need to be careful with coincidence limits of derivatives. The corresponding Green function associated with the Laplacian can then be expanded as

$$\tilde{\Delta}^{-1}(\xi, \xi')^{ij} = -\int \frac{\mathrm{d}s}{(4\pi s)^{N/2}} \exp\left(-\frac{\sigma(\xi, \xi')}{2s}\right) \sum_{l=0}^{\infty} s^l a_l^{ij}(\xi, \xi'). \tag{35}$$

Note that the trace over fluctuations in the range $k_{\min}, k_{\max}$ is implemented by integrating s over the range, $k_{\max}^{-2}, k_{\min}^{-2}$. The heat kernel can be exploited to regularize the trace in the problem. For instance, it can be written in the coincidence limit

$$\mathrm{Tr} \log(-\tilde{\Delta}) = -\int \mathrm{d}A \int \frac{\mathrm{d}s}{s(4\pi s)^{N/2}} \sum_{l=0}^{\infty} s^l a_l^{ii}(\xi), \tag{36}$$

and the following expression of great use below:

$$\Delta^{-2}(\xi, \xi')^{ij} = \int \frac{\mathrm{d}s \quad s}{(4\pi s)^{N/2}} \exp\left(-\frac{\sigma}{2s}\right) \sum_{l=0}^{\infty} s^l a_l(\xi, \xi')^{ij}. \tag{37}$$

All this allows us to calculate the second term in equation (3). So, taking the logarithm of equation (25) and expanding we find

$$\mathrm{Tr}\log(\mathcal{L}^{ij}) \simeq \mathrm{Tr}\log\left(\kappa\,\tilde{\Delta}^2_{ij}\right) + \frac{1}{\kappa}\,\mathrm{Tr}\,A^{abik}(\tilde{\nabla}_a\tilde{\nabla}_b)_{k\ell}(\tilde{\Delta}^{-2})^{\ell j} + \frac{1}{\kappa}\,\mathrm{Tr}\,B^{aik}\tilde{\nabla}_{ak\ell}(\tilde{\Delta}^{-2})^{\ell j} + \frac{1}{\kappa}\,\mathrm{Tr}\,C_{ik}(\tilde{\Delta}^{-2})^{kj}. \qquad (38)$$

As mentioned in section 1, the contribution of the measure is still to be taken into account. To order one loop, that is given by minus the first term in equation (38), so its contribution cancels out the first term [27].

By using the expression for the operator of fluctuations in equation (26), it is easy to show that the last two terms are at least of the order K^4, and therefore can just be dropped. Therefore, the only contribution for the renormalization comes from the second term. By taking two derivatives of equation (37), we get that in the coincidence limit, $\xi' \to \xi$,

$$\begin{aligned}\mathrm{Tr}\,A^{ab}\tilde{\nabla}_a\tilde{\nabla}_b\tilde{\Delta}^{-2} &= \sum_{l=0}^{\infty}\int\frac{\mathrm{d}s\,s^{l+1}}{(4\pi s)^{N/2}}\int\mathrm{d}A\,A^{ab}\tilde{\nabla}_a\tilde{\nabla}_b\left\{a_l\exp\left(-\frac{\sigma}{2s}\right)\right\}\\ &= \sum_{l=0}^{\infty}\int\frac{\mathrm{d}s\,s^{l}}{(4\pi s)^{N/2}}\int\mathrm{d}A\,A^{abij}\left(s\nabla_a\nabla_b-\frac{\gamma_{ab}}{2}\right)a_l^{ij}.\end{aligned} \qquad (39)$$

The second line follows because, in the coincidence limit, only the term appearing from differentiating σ twice survives:

$$\tilde{\nabla}_a\tilde{\nabla}_b\left(a_l\exp\left(-\frac{\sigma}{2s}\right)\right) = \tilde{\nabla}_a\tilde{\nabla}_b a_l - \frac{\gamma_{ab}}{2s}a_l. \qquad (40)$$

From equations (39) and (26) we obtain the one-loop effective bendings for the Helfrich membrane for arbitrary codimension. For $N = 2$ (i.e., for a two-dimensional membrane) only the terms with $l = 0$ must be taken into account to renormalize any ultraviolet singularity, $s \to 0$. Moreover, the Van Vleck–Morette determinant does not contribute at all. In addition, from equation (26) we see that

$$A^{a}{}_{ajj} = \frac{\kappa}{2}(2(2+D))K^2 - 2\mu D - 4\kappa\mathcal{R}, \qquad (41)$$

for a two-dimensional membrane embedded into a $(2 + D)$-dimensional Euclidean space. Using this result in equation (39) we find the effective bending coupling which describes the crumpling transition, see, e.g., [8]. For the rigidity we have

$$\kappa_{\mathrm{eff}} = \kappa - T\frac{2+D}{4\pi}\log\frac{L}{a}, \qquad (42)$$

where L is the linear size of the membrane and a is a microscopic cutoff. If the codimension is set to $D = 1$, this equation reproduces precisely the result already found in [12–15]. In the string theory context, and using a different approach, this result was found in [25, 26]. For the surface tension we obtain

$$\mu_{\mathrm{eff}} = \mu + T\frac{D}{4\pi}\frac{\mu}{\kappa}\log\frac{L}{a}. \qquad (43)$$

A result which coincides with that found in [14] (also for arbitrary codimension) and with that from [32] (obtained for $D = 1$). This method provides a way to identify the β_{eff} parameter also for arbitrary D. For a two-dimensional membrane we get

$$\beta_{\mathrm{eff}} = \beta + \frac{2T}{4\pi}\log\frac{L}{a}, \qquad (44)$$

which is also in agreement with the result found in [14] for $D = 1$ and, as expected, the renormalization of this β coefficient does not depend on D. Note that if the contribution of the measure is not taken into account, the term $\mathrm{Tr}\log(-\tilde{\Delta})$ gives a spurious one-loop contribution to the renormalization of both μ_{eff} and β_{eff}. Moreover, for a two-dimensional membrane the Gaussian term does not give a contribution as it should because in this case the term is a topological invariant.

5. Summary

In this paper we have used an extended covariant geometric approach recently developed to obtain the one-loop effective bending couplings of a geometric Hamiltonian associated with a two-dimensional membrane embedded into a $(2 + D)$-dimensional space. Our result is in agreement with that appearing in the literature for a two-dimensional membrane embedded into a three-dimensional Euclidean space. Work on the two-loop calculation is now underway.

Acknowledgments

One of us, JAS, is grateful to Jemal Guven for many discussions and to Professor H Kleinert for kind hospitality at the Institute für Theoretische Physik, Freie Universität Berlin. AZ acknowledges hospitality of the Instituto de Ciencias Nucleares, UNAM. This work was supported by the PROMEP program PTC in Mexico.

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